

Applied Numerical Mathematics Computer Project 4

Solve the boundary value problem

$$(1 - c^2)u'' + \varepsilon u'''' = \sin u$$

with boundary conditions

$$\lim_{x \rightarrow \infty} u(x) = 2\pi \quad \lim_{x \rightarrow -\infty} u(x) = 0 \quad \lim_{x \rightarrow \pm\infty} u'(x) = 0$$

and $|c| < 1$. For $\varepsilon = 0$, the exact solution is given by

$$u(x) = 4 \tan^{-1} \left(\exp \left[\frac{x - x_0}{\sqrt{1 - c^2}} \right] \right).$$

Compute numerical solutions for $0 < \varepsilon \ll 1$, and discuss the results.