Lecture on March 30, 2009

Math Modeling I

Comments on Project I

- Except one report, all received reports show a good level of understanding
  - Statement of the question
  - Model construction
    - Coordinates transformation (2D ←→ 3D)
    - Assignment of values on the 2D plane
    - Discussion on interpolation techniques
  - Experiments (model verification)
    - Data sets
    - Show image outputs
- Improvements
  - Implementation ("just do it", Matlab programming)
  - 3D geometry (coordinate systems)

Sample Report 1

- Good write-up
- Good code
- Experiment with real data
- Needs more experiments on various data sets

Linear Algebra

- View a matrix as an "array of columns": $A_{mxn} = [\mathbf{a}_1,\mathbf{a}_2,\ldots,\mathbf{a}_n]$ with $\mathbf{a}_i=[a_{1i},a_{2i},\ldots,a_{mi}]^T$
- If $x=[x_1,x_2,\ldots,x_n]^T$, then Ax can be viewed as a linear combination of the columns of A: $Ax = x_1\mathbf{a}_1+x_2\mathbf{a}_2+\ldots+x_n\mathbf{a}_n$
- A matrix is symmetric if $A=A^T$
- A matrix is orthogonal if its columns are orthogonal vectors: $\mathbf{a}_i^T\mathbf{a}_j=0$ for all $i \neq j$; $\mathbf{a}_i^T\mathbf{a}_i=1$

Important Theorems

- Theorem 1: If A_{mxn} is an orthogonal matrix ($m\geq n$), then $A^TA=I_{nxn}$
- Proof. Let $A = [\mathbf{a}_1,\mathbf{a}_2,\ldots,\mathbf{a}_n]$. Since A is orthogonal, $\mathbf{a}_i^T\mathbf{a}_j=0$ for all $i\neq j$; $\mathbf{a}_i^T\mathbf{a}_i=1$
  Now, the (i,j) entry of $A^TA=\mathbf{a}_i^T\mathbf{a}_j=\delta_{ij}$.
  So, $A^TA=I$.
- Theorem 2. A^TA and AA^T are symmetric matrices.

Important Theorems, cont'd

- Theorem 3. A matrix is symmetric if and only if it is orthogonally diagonalizable.
- $A=A^T \leftrightarrow A=UDU^T$, U orthogonal, D diagonal
- Eigenvalue and eigenvector: λ is an eigenvalue of A if for some vector $v\neq 0$,
  $Av= \lambda v$.
  We call v an eigenvector of A associated with eigenvalue λ.
- Theorem 4. A symmetric matrix is diagonalized by a matrix of its orthogonal eigenvectors.

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